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Seesaw induced electroweak scale, the hierarchy problem and sub-eV neutrino masses

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Abstract. We describe a model for the scalar sector where all interactions occur either at an ultra-high scale, $\Lambda_{\rm U} \sim 10^{16}$–$10^{19}$ GeV, or at an intermediate scale, $\Lambda_{\rm I} = 10^{9}$–10^{11} GeV. The interaction of physics on these two scales results in an $SU(2)$ Higgs condensate at the electroweak (EW) scale, $\Lambda_{\rm EW}$, through a seesaw-like Higgs mechanism, $\Lambda_{\rm EW} \sim \Lambda_{\rm I}^2/\Lambda_{\rm U}$, while the breaking of the SM $SU(2)\times U(1)$ gauge symmetry occurs at the intermediate scale $\Lambda_{\rm I}$. The EW scale is, therefore, not fundamental but is naturally generated in terms of ultra-high energy phenomena and so the hierarchy problem is alleviated. We show that the class of such "seesaw Higgs" models predict the existence of sub-eV neutrino masses which are generated through a "two-step" seesaw mechanism in terms of the same two ultra-high scales: $m_\nu \sim \Lambda_{\rm I}^4/\Lambda_{\rm U}^3 \sim \Lambda_{\rm EW}^2/\Lambda_{\rm U}$. The neutrinos can be either Dirac or Majorana, depending on the structure of the scalar potential. We also show that our seesaw Higgs model can be naturally embedded in theories with *tiny* extra dimensions of size $R \sim \Lambda_{\rm U}^{-1} \sim 10^{-16}$ fm, where the seesaw induced EW scale arises from a violation of a symmetry at a distant brane; in particular, in the scenario presented there are seven *tiny* extra dimensions.

1 Introduction

A long standing problem in modern particle physics is the apparent enormous hierarchies of energy/mass scales observed in nature. Disregarding the "small" hierarchies in the masses of the known charged matter particles, there seem to be two much larger hierarchies: the first is the hierarchy between the fundamental grand unified scale $\Lambda_{\rm U} \sim \mathcal{O}(10^{16})$ GeV [or Planck scale $\Lambda_{\rm U} \sim \mathcal{O}(10^{19})$ GeV], and the EW scale, $\Lambda_{\rm EW} \sim \mathcal{O}(100)$ GeV, and the second is the hierarchy between the EW scale and the neutrino mass scale $m_\nu \sim \mathcal{O}(10^{-3})$ eV. Denoting the former as the upper hierarchy (UH) and the latter as the lower hierarchy (LH), it is interesting to note that these two hierarchies (the UH and LH) span over roughly the same energy scales, i.e., over about 14 orders of magnitude. This hierarchical structure of scales raises some unavoidable questions: What is the physics behind EW symmetry breaking (EWSB)? Is it the same physics that underlies the neutrino mass scale? Does the scale of the new physics lie close to the EW scale or close to the GUT or Planck scale? And finally, is there a theory that can provide a natural explanation for the simultaneous presence of $\Lambda_{\rm U}$, $\Lambda_{\rm EW}$ and m_ν? These questions have fueled a lot of activity in the past 30 years in the search for new physics beyond the standard model (SM).

An example of a successful mechanism for explaining the LH (i.e., $m_\nu/\Lambda_{\rm EW}$) was suggested a long time ago [1] – the so called seesaw mechanism in which the new physics is assumed to lie at the ultra-high fundamental scale $\Lambda_{\rm U}$. Given the EW scale, this mechanism utilizes the UH (i.e., $\Lambda_{\rm EW}/\Lambda_{\rm U}$) to solve the LH, since the seesaw generated neutrino mass scale is $m_\nu \sim (\Lambda_{\rm EW}/\Lambda_{\rm U}) \times \Lambda_{\rm EW}$.

The UH, when viewed within the SM framework, is often called the hierarchy problem (HP) of the SM, which is intimately related to the SM Higgs sector responsible for the generation of the EWSB scale, $v_{\rm EW} \sim \Lambda_{\rm EW}$, through the Higgs mechanism. The HP of the SM raises a technical difficulty known as the naturalness (or fine tuning) problem. Simply put, the presence of a fundamental EW scale seems to be unnatural, since if the only physics which exists up to some ultra-high grand unified scale $\Lambda_{\rm U}$ is the SM, then it is hard to see why the Higgs boson does not receive large corrections to its mass. In other words, there is a problem of stabilizing the $\mathcal{O}(\Lambda_{\rm EW})$ mass scale of the Higgs against radiative corrections without an extreme fine tuning (to one part in $\Lambda_{\rm EW}^2/\Lambda_{\rm U}^2$) between the bare Higgs mass squared and the $\mathcal{O}(\Lambda_{\rm U}^2)$ corrections, $\Lambda_{\rm U}$ being the grand unified or Planck scale.

This fine tuning problem of the Higgs mass term has been addressed by numerous models for physics beyond the SM. For instance, in weak scale supersymmetry (SUSY) models, in which the SUSY breaking scale $M_{\rm SUSY}$ is close to the EW scale, such large corrections to the Higgs mass are suppressed, due to the symmetry between fermions and

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bosons giving rise to natural cancellations between graphs involving particles and superparticles [2]. Likewise, in technicolor inspired models, new gauge interactions dynamically generate a new scale $M_{\rm dyn}$ from which the EWSB is generated. In this scenario the light Higgs is usually viewed as a pseudo-Goldstone boson associated with the breaking of the new strong gauge force and the HP is alleviated if $M_{\rm dyn}$ is close to $\Lambda_{\rm EW}$. In this case, somewhat like in the SUSY case, it has been recently argued that one may be able to explain away the little fine tuning left (i.e., between $M_{\rm dyn}$ and $\Lambda_{\rm EW}$) by "little Higgs" models [3], through loops involving additional particles with TeV scale masses that cancel the SM loop graphs. Contrary to SUSY models, in little Higgs models these cancellations occur between particles of the same statistics.

Another interesting approach for addressing the fine tuning problem of the SM was suggested in [4]. According to [4], the scale invariance of the classical level SM can be used to remove the explicit quadratic divergences in the higher order corrections to the Higgs mass order by order in perturbation theory, without invoking any fine tuning. In this case, both the bare Higgs mass and the Higgs mass counter term can be chosen at the EW scale and so the conditions on the Higgs mass are natural.

However, SUSY and little Higgs models as well as the scale invariance theorem do not provide a solution to the question of the origin of scales, i.e., why do we observe such a large hierarchy between the fundamental GUT or Planck scale $\Lambda_{\rm U}$ and the EW scale $\Lambda_{\rm EW}$ (recall that $M_{\rm SUSY} \sim \Lambda_{\rm EW}$ in the SUSY case and $M_{\rm dyn} \sim \Lambda_{\rm EW}$ in the technicolor-like models case)? Of course, these models do give a successful resolution to the technical difficulty of explaining the simultaneous presence of the two disparate scales $\Lambda_{\rm U}$ and $\Lambda_{\rm EW}$, thus solving the naturalness or fine tuning problem. In contrast, in extra-dimensional scenarios such questions are muted since these theories contain only one fundamental scale and so the hierarchy between the Planck and the EW scales is simply absent. For example, in large extra-dimensional models [5] the EWSB scale $\Lambda_{\rm EW}$ is considered to be the only fundamental scale and the observed enormity of the Planck scale is a consequence of the large extra dimensions through which gravity propagates. Alternatively, in models where the four-dimensional metric is multiplied by a "warp" exponential (rapidly changing) factor of one tiny extra dimension [6], the Planck scale is viewed as the fundamental scale and the EW mass scale is generated due to this exponential suppression factor.

In this paper we propose an alternative approach, where the only fundamental scale is the GUT or Planck scale $\Lambda_{\rm U}$, while the EW and neutrino mass scales both arise due to interactions between the fundamental scale $\Lambda_{\rm U}$ and a new intermediate ultra-high scale $\Lambda_{\rm I} \sim 10^9$–$10^{11}$ GeV, i.e., $\Lambda_{\rm EW} << \Lambda_{\rm I} << \Lambda_{\rm U}$. The intermediate scale is viewed as the scale of breaking of the unification group which underlies the physics at the scale $\Lambda_{\rm U}$ (see e.g., [7]). In particular, this class of models (see also [8]) are based on the idea that EWSB occurs at $\Lambda_{\rm I}$ whereas the smallness of the EW mass scale is a consequence of a $\Lambda_{\rm U} - \Lambda_{\rm I}$ seesaw-like Higgs mechanism in the scalar potential, giving $\Lambda_{\rm EW} \sim \Lambda_{\rm I}^2/\Lambda_{\rm U}$, from which the masses of all known particles are generated. In addition, our model naturally accounts for the existence of sub-eV neutrino masses by means of a "two-step" or "double" seesaw mechanism using the same two ultra-high scales $\Lambda_{\rm U}$ and $\Lambda_{\rm I}$: the first $\Lambda_{\rm U}$–$\Lambda_{\rm I}$ seesaw generates the EW scale $\Lambda_{\rm EW} \sim \Lambda_{\rm I}^2/\Lambda_{\rm U}$ and then a second $\Lambda_{\rm U}$–$\Lambda_{\rm EW}$ seesaw gives rise to the sub-eV neutrino mass scale $m_\nu \sim \Lambda_{\rm EW}^2/\Lambda_{\rm U} \sim \Lambda_{\rm I}^4/\Lambda_{\rm U}^3$. Thus, in this type of "seesaw Higgs" models the large desert (gap) between the fundamental scale $\Lambda_{\rm U} \sim 10^{16}$ or 10^{19} GeV and the EW scale $\Lambda_{\rm EW}$ (the UH) and between the EW scale and the neutrino mass scale (the LH) are both naturally explained by introducing an intermediate ultra-high scale, $\Lambda_{\rm I} \sim 10^9$–$10^{11}$ GeV, in which new interactions are manifested *only in the scalar sector*.

We will present one possible minimal variation of seesaw Higgs models which we will name the "light seesaw" model. In our light seesaw model the scalar spectrum consists of several physical scalars with masses of order $\Lambda_{\rm U}$ and one SM-like Higgs with a mass of $\mathcal{O}(\Lambda_{\rm EW})$. The light seesaw model will be minimally constructed in the sense that at energy scales of $\mathcal{O}(\Lambda_{\rm I})$ and below, i.e., much smaller than the fundamental scale $\Lambda_{\rm U}$, it contains the SM gauge symmetries and fields apart from the addition of new (superheavy) right-handed neutrino fields. Thus, at energies of $\mathcal{O}(\Lambda_{\rm EW})$, the light seesaw model which may be considered as a minimal extension of the SM without a HP, will have the same phenomenological signatures as the "one-Higgs" SM. This implies that, at the LHC, only one SM-like Higgs state will be observed in contrast to e.g. the expectations from SUSY models in which several Higgs states should be detected. In addition, due to the presence of the superheavy right-handed neutrinos, our light seesaw model will contain sub-eV neutrino masses in accord with recent measurements [9].

We also show that seesaw Higgs models can emanate from theories with extra spatial dimensions. However, contrary to large extra dimensions models in which the $4+n$ fundamental Planck scale (n is the number of extra dimensions) is taken to be of $\mathcal{O}(\Lambda_{\rm EW})$, the seesaw Higgs mechanism requires the fundamental scale to be of $\mathcal{O}(\Lambda_{\rm U})$, which results in *tiny* compact extra dimensions of size $R \sim \Lambda_{\rm U}^{-1}$. This somewhat resembles the "warp" extra dimension model of [6] which also requires a tiny extra dimension with a fundamental scale of order of the Planck scale. We present a "tiny extra dimensions" scenario that can naturally explain the existence of an intermediate scale $\Lambda_{\rm I} \sim 10^9$ GeV, required for triggering the seesaw Higgs mechanism. In particular, in this scenario the intermediate scale $\Lambda_{\rm I}$ is generated due to violation of some symmetry at a distant brane.

We wish to emphasize that the light seesaw Higgs model presented in this paper does not attempt to represent the complete theory but, only to parametrize its low energy dynamics and to provide a schematic model that holds the key ingredients for model building of a more complete underlying seesaw Higgs model.

Finally, we note that a scalar model that falls into the category of light seesaw models was also proposed by

Calmet in [8]. The Calmet model is re-examined in this paper since it demonstrates the seesaw Higgs mechanism with only $SU(2)$ scalar doublets, whereas in our light seesaw model the seesaw Higgs mechanism is based on interactions between $SU(2)$ doublets and singlets. Moreover, the issue of neutrino masses was not addressed in [8]. It turns out that the Calmet model is complimentary to ours since it predicts Dirac neutrinos whereas our model gives rise to Majorana neutrinos. We also note that the idea of a seesaw mechanism which originates from a scalar sector was applied before in the so called "top quark seesaw models", in which the EWSB is triggered by a condensation which appears due to strong topcolor gauge interactions. In these models, the right size of the top quark mass is ensured by a seesaw structure in the masses of the composite states [10]. A seesaw mechanism in the scalar sector was also applied to SUSY models as a possible solution to the μ-problem [11].

The paper is organized as follows: in Sect. 2 we describe the seesaw Higgs mechanism as it is manifested in the light seesaw model. In Sect. 3 we discuss the generation of neutrino masses in seesaw Higgs models, in Sect. 4 we discuss the possibility of embedding the light seesaw model in theories with extra dimensions and in Sect. 5 we summarize our findings.

2 The seesaw Higgs mechanism

As mentioned above, in the seesaw Higgs models presented below, the EW scale $\Lambda_{\rm EW}$ is not fundamental since its existence is triggered by physics at much higher energy scales (and therefore more fundamental). In particular, $\Lambda_{\rm EW}$ is generated by the large splitting between the fundamental scale $\Lambda_{\rm U}$ and an intermediate scale $\Lambda_{\rm I}$, such that

$$\epsilon \equiv \Lambda_{\rm I}/\Lambda_{\rm U} \sim 10^{-7} \text{ or } 10^{-8.5}, \tag{1}$$

depending on whether the fundamental scale $\Lambda_{\rm U}$ is taken to be around the GUT scale [$\Lambda_{\rm U} \sim \mathcal{O}(10^{16})$ GeV] or around the Planck scale [$\Lambda_{\rm U} \sim \mathcal{O}(10^{19})$ GeV], respectively.

Let us schematically define the seesaw Higgs total Lagrangian as

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\rm SM}(f,G) + \mathcal{L}_{\rm S}(\Phi,S) \\ +\mathcal{L}_{\rm Y}(\Phi,f) + \mathcal{L}_{\nu}(\Phi,S,\nu_{\rm R}),\end{aligned} \tag{2}$$

where $\mathcal{L}_{\rm SM}(f,G)$ is the usual SM's fermions and gauge bosons kinetic terms, $\mathcal{L}_{\rm S}(\Phi,S)$ contains the seesaw-like scalar potential as well as the kinetic terms for the $SU(2)$ doublet Φ and the scalar singlets S, $\mathcal{L}_{\rm Y}(\Phi,f)$ is the SM Yukawa terms and $\mathcal{L}_{\nu}(\Phi,S,\nu_{\rm R})$ contains both Dirac- and Majorana-like interactions between the scalars and the left and right-handed neutrinos ($\nu_{\rm R}$). In what follows we will consider two types of $\mathcal{L}_{\rm S}(\Phi,S)$ that can give rise to a seesaw Higgs mechanism; one that leads to our light seesaw model and another that leads to the seesaw Higgs model discussed in [8] – the "Calmet model".

2.1 The light seesaw model

Consider the following scalar Lagrangian:

$$\begin{aligned}&\mathcal{L}_{\rm S}(\Phi,\varphi,\chi) \\ &= |D_\mu\Phi|^2 + |\partial_\mu\varphi|^2 + |\partial_\mu\chi|^2 - V(\Phi,\varphi,\chi),\end{aligned} \tag{3}$$

where Φ is an $SU(2)$ doublet and φ, χ are "sterile" $SU(2)$ singlets which do not interact with the SM fields. A scalar potential containing the two singlets and one doublet in (3) and subject to a seesaw Higgs mechanism can be constructed as follows:

$$\begin{aligned}V(\Phi,\varphi,\chi) = &\lambda_1\left(|\Phi|^2 - |\chi|^2\right)^2 + \lambda_2\left(|\varphi|^2 - \Lambda_{\rm U}^2\right)^2 \\ &+ \lambda_3\left(\mathrm{Re}(\varphi^\dagger\chi) - \Lambda_{\rm I}^2\cos\xi\right)^2 \\ &+ \lambda_4\left(\mathrm{Im}(\varphi^\dagger\chi) - \Lambda_{\rm I}^2\sin\xi\right)^2,\end{aligned} \tag{4}$$

where, we have assumed a massless $SU(2)$ doublet field Φ, one massless singlet field χ and one superheavy singlet field φ.[1] Also, all λ_i are positive real constants, naturally of $\mathcal{O}(1)$, and ξ is a possible relative phase between the VEVs of φ and χ which may give rise to spontaneous CP-violating effects, e.g., in the neutrino sector. Note that the total light seesaw Lagrangian (including the neutrino Yukawa interaction terms) conserves $U(1)_L$, where L is the lepton number, if we asign the scalar singlets lepton number 2, i.e., $L_\varphi = L_\chi = 2$ (see discussion in the next section).

The light seesaw potential in (4) gives rise to the desired seesaw condensate of Φ by "coupling" it [through the term $\lambda_1(|\Phi|^2-|\chi|^2)^2$] to a seesaw induced VEV of the singlet field χ. In particular, the minimization of $V(\Phi,\varphi,\chi)$ (assuming CP-conservation, thus setting $\xi=0$) which only contains terms at energy scales $\Lambda_{\rm U}$ and $\Lambda_{\rm I}$ leads to

$$\begin{aligned}&\langle\varphi\rangle = \Lambda_{\rm U}, \\ &\langle\Phi\rangle = \langle\chi\rangle = \frac{\Lambda_{\rm I}^2}{\Lambda_{\rm U}} \equiv v_{\rm EW} \sim \Lambda_{\rm EW},\end{aligned} \tag{5}$$

where $\langle\Phi\rangle = v_{\rm EW}$ is the condensate required for EWSB.

After EWSB (by $\langle\Phi\rangle = v_{\rm EW}$) the W and Z gauge bosons acquire their masses in the usual way by "eating" the non-physical charged and neutral Goldstone fields of the $SU(2)$ doublet. We are then left with three CP-even and two CP-odd physical neutral scalar states. Expanding around the vacuum, using (5) with $v_{\rm EW} = \langle\Phi\rangle = \langle\chi\rangle$ and $v_\varphi = \langle\varphi\rangle$, we can write

$$\begin{aligned}&\Phi = \begin{pmatrix} 0 \\ v_{\rm EW} + h_\Phi \end{pmatrix}, \\ &\varphi = v_\varphi + h_\varphi + \mathrm{i}a_\varphi, \\ &\chi = v_{\rm EW} + h_\chi + \mathrm{i}a_\chi.\end{aligned} \tag{6}$$

[1] The massless $SU(2)$ doublet Φ and singlet χ may be Goldstone bosons related to a spontaneously broken global symmetry at the high scale $\Lambda_{\rm U}$.

Then, diagonalizing the scalar mass matrix we obtain the physical states which are (up to corrections smaller than or of $\mathcal{O}(\epsilon^2)$):

$$\begin{aligned} CP\text{-even}: &\ H \approx h_\Phi,\ S_1 \approx h_\chi,\ S_2 \approx h_\varphi, \\ CP\text{-odd}: &\ A_1 \approx a_\chi,\ A_2 \approx a_\varphi, \end{aligned} \quad (7)$$

with masses (up to small corrections of $\mathcal{O}(\epsilon^4)$)

$$\begin{aligned} M_H &\sim 2\sqrt{\lambda_1}\frac{\Lambda_{\rm I}^2}{\Lambda_{\rm U}} = 2\sqrt{\lambda_1} v_{\rm EW}, \\ M_{S_1} &\sim \sqrt{\lambda_3}\Lambda_{\rm U}, \\ M_{S_2} &\sim 2\sqrt{\lambda_2}\Lambda_{\rm U}, \\ M_{A_1} &\sim \sqrt{\lambda_4}\Lambda_{\rm U}, \\ M_{A_2} &= 0. \end{aligned} \quad (8)$$

Thus, the SM-like Higgs, H, acquires a mass of order $\Lambda_{\rm EW}$ while the two CP-even physical singlets S_1 and S_2 become superheavy, i.e., with masses of the order of the fundamental scale $\Lambda_{\rm U}$. Also, there is one superheavy axial singlet A_1, and a massless axial state A_2 which is the majoron associated with the spontaneous breakdown of lepton number if $L_\varphi = L_\chi = 2$; see the next section. Thus, at energies of $\mathcal{O}(\Lambda_{\rm EW})$, the light seesaw model reproduces the one-light-Higgs SM.

2.2 The Calmet model

An alternative approach that falls into the category of seesaw Higgs models was suggested by Calmet in [8]. Calmet's model is simply a two Higgs doublet model with a seesaw Higgs mass matrix embedded in the scalar potential:

$$\begin{aligned} V(\Phi_1\Phi_2) &= \left(\Phi_1^\dagger \Phi_2^\dagger\right)\begin{pmatrix} 0 & \Lambda_{\rm I}^2 \\ \Lambda_{\rm I}^2 & \Lambda_{\rm U}^2 \end{pmatrix}\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \\ &\quad + \lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2, \end{aligned} \quad (9)$$

where one assumes a massless doublet Φ_1 (see footnote 1) and a superheavy second doublet Φ_2, with a mixing interaction term proportional to the intermediate high scale $\Lambda_{\rm I}$. Minimizing the Calmet's potential in (9) one finds

$$\begin{aligned} \langle\Phi_1\rangle &\sim \frac{1}{\sqrt{2\lambda_1}}\frac{\Lambda_{\rm I}^2}{\Lambda_{\rm U}} \equiv v_{\rm EW}, \\ \langle\Phi_2\rangle &\sim -\frac{1}{\sqrt{2\lambda_1}}\frac{\Lambda_{\rm I}^4}{\Lambda_{\rm U}^3}. \end{aligned} \quad (10)$$

Thus, here also the EWSB is triggered by the $\Lambda_{\rm U}$–$\Lambda_{\rm I}$ seesaw induced VEV of the light $SU(2)$ doublet $v_{\rm EW} = \langle\Phi_1\rangle$, while the second supermassive doublet acquires a tiny – "double seesawed" – VEV, $\langle\Phi_2\rangle \sim \mathcal{O}(\epsilon^2 v_{\rm EW})$, which, as will be shown in the next section, may be responsible for generating the sub-eV neutrino mass scale. Like any other two Higgs doublets model, after EWSB the physical scalar spectrum in the Calmet model consists of three neutral scalars (two CP-even and one CP-odd) and two charged Higgs states. In particular, one light CP-even SM-like Higgs h with $M_h \sim 2\sqrt{\lambda_1} v_{\rm EW} \sim \mathcal{O}(\Lambda_{\rm EW})$, and four superheavy Higgs states H, A and $H^\pm$ [i.e., two neutral CP-even (H) and CP-odd (A) ones and two charged ones ($H^\pm$)], which form the supermassive $SU(2)$ doublet Φ_2 and, therefore, acquire $\mathcal{O}(\Lambda_{\rm U})$ masses. The supermassive scalar states decouple from the model at energy scales much smaller than the fundamental scale $\Lambda_{\rm U}$. Thus, similar to our light seesaw model, the Calmet's model also reproduces the one-light-Higgs SM at the EW scale.

3 Neutrino masses

Introducing right-handed neutrinos, the neutrino scalar Yukawa Lagrangian in our light seesaw model takes the form:[2]

$$\mathcal{L}_\nu = -Y_{\rm D}\ell_{\rm L}\Phi\nu_{\rm R} + Y_{\rm M}\varphi\bar{\nu_{\rm R}^{\rm c}}\nu_{\rm R} + \text{h.c.}, \quad (11)$$

where $\ell_{\rm L}$ is the left handed $SU(2)$ lepton doublet, $\nu_{\rm R}$ is the right-handed neutrino field, Φ and φ are the $SU(2)$ scalar doublet and singlet, respectively, and $Y_{\rm D}$, $Y_{\rm M}$ are the usual Dirac- and Majorana-like Yukawa couplings.

Thus, when the singlet φ forms the condensate $\langle\varphi\rangle = \Lambda_{\rm U}$ the second term in (11) will lead to a right-handed Majorana mass which will naturally be of that order: $m_\nu^{\rm M} = Y_{\rm M}\Lambda_{\rm U}$. The $SU(2)$ condensate $\langle\Phi\rangle = \Lambda_{\rm I}^2/\Lambda_{\rm U} \sim \Lambda_{\rm EW}$ will generate a Dirac mass for the neutrinos of size $m_\nu^{\rm D} \sim Y_{\rm D}\Lambda_{\rm EW}$ through the first term in (11). Then, the neutrino mass matrix becomes

$$\mathcal{L}_{m_\nu} = \left(\bar{\nu_{\rm L}^{\rm c}}\bar{\nu_{\rm R}}\right)\begin{pmatrix} 0 & m_\nu^{\rm D} \\ m_\nu^{\rm D} & m_\nu^{\rm M} \end{pmatrix}\begin{pmatrix} \nu_{\rm L} \\ \nu_{\rm R}^{\rm c} \end{pmatrix}, \quad (12)$$

which, upon diagonalization (i.e., the classic seesaw mechanism) gives two Majorana neutrino physical states: a superheavy state ν_h with a mass $m_{\nu_h} \sim Y_{\rm M}\Lambda_{\rm U}$ and a superlight state ν_ℓ with a mass

$$m_{\nu_\ell} = \frac{(m_\nu^{\rm D})^2}{m_\nu^{\rm M}} = \frac{Y_{\rm D}^2}{Y_{\rm M}}\frac{\Lambda_{\rm I}^4}{\Lambda_{\rm U}^3} \sim \frac{Y_{\rm D}^2}{Y_{\rm M}}\frac{\Lambda_{\rm EW}^2}{\Lambda_{\rm U}}. \quad (13)$$

The neutrino mass scale is, therefore, subject to a two-step seesaw mechanism, the first (in the scalar sector) generates the Dirac neutrino mass $m_\nu^{\rm D} \sim Y_{\rm D}\Lambda_{\rm EW}$, which then enters in the off diagonal neutrino mass matrix to give the classic seesaw Majorana mass in (13) by a second $m_\nu^{\rm M}$–$m_\nu^{\rm D}$ seesaw in the neutrino mass matrix of (12). The presence of this extremely small scale, $m_{\nu_\ell} \sim \mathcal{O}(\Lambda_{\rm EW}^2/\Lambda_{\rm U})$, well below the EW scale, is therefore naturally explained in terms of the two ultra-high scales $\Lambda_{\rm U}$ and $\Lambda_{\rm I}$.

As an example, let us suppose that $\Lambda_{\rm U} \sim \mathcal{O}(10^{16})$ GeV. As was shown in the previous section, this means that

[2] Note that the second singlet χ can also couple to the right-handed neutrinos via $\chi\bar{\nu_{\rm R}^{\rm c}}\nu_{\rm R}$. However, since χ forms a condensate of $\mathcal{O}(\Lambda_{\rm EW})$, its contribution to the Majorana neutrino mass term will be negligible compared to that of φ which forms the condensate of $\mathcal{O}(\Lambda_{\rm U})$.

$\Lambda_{\rm I} \sim \mathcal{O}(10^9)$ will be needed in order to generate $\Lambda_{\rm EW} = \mathcal{O}(100)\,{\rm GeV}$. Then, if $Y_{\rm D} \sim Y_{\rm M} \sim \mathcal{O}(1)$, it follows from (13) that $m_\nu = \mathcal{O}(10^{-3})\,{\rm eV}$, roughly in accord with current mixing results [9]. A value of $\Lambda_{\rm U}$ at the Planck scale could still be consistent with the double seesaw sub-eV neutrino masses. In particular, if $Y_{\rm D}^2/Y_{\rm M} \sim \mathcal{O}(10^3)\,{\rm GeV}$, then with $\Lambda_{\rm I} = \mathcal{O}(10^{10.5})\,{\rm GeV}$ [which gives $\Lambda_{\rm EW} = \mathcal{O}(100)\,{\rm GeV}$ when $\Lambda_{\rm U} \sim \mathcal{O}(10^{19})\,{\rm GeV}$], we still obtain $m_\nu = \mathcal{O}(10^{-3})\,{\rm eV}$. This may happen if either the Majorana mass is sufficiently below the Planck scale, i.e., $Y_{\rm M} \sim \mathcal{O}(10^{-3})$, and the Dirac masses are of the order of the EW scale, i.e., $Y_{\rm D} \sim \mathcal{O}(1)$, or if the Majorana mass is of the order of the intermediate scale $\Lambda_{\rm I}$, i.e., $Y_{\rm M} \sim \mathcal{O}(10^{-8.5})$ and the Dirac masses are of $\mathcal{O}(100)\,{\rm MeV}$ (consistent with most light leptons and down quark masses) which corresponds to $Y_{\rm D} \sim \mathcal{O}(10^{-3})$. The latter possibility, in which $m_{\nu_h} \sim \Lambda_{\rm I} \sim \mathcal{O}(10^{10.5})$, is particularly interesting since the existence of heavy Majorana neutrinos with masses in the range 10^9–$10^{13}\,{\rm GeV}$ may be useful for leptogenesis, i.e., for generating the observed baryon asymmetry through the lepton asymmetry which is triggered by the decays of these heavy Majorana neutrinos [12].

As was noted in the previous section, our light seesaw model contains a majoron which is the massless Goldstone boson (i.e., the axial component of the singlet field φ) associated with the spontaneous breaking of a global $U(1)$ number [13]. In particular, in our case, the model defined by the total Lagrangian in (2) with the scalar potential in (4) and with the neutrino Yukawa terms in (11), conserves lepton number L if both singlets φ and χ carry lepton number 2, i.e., if $L_\varphi = L_\chi = 2$. Thus, when φ and χ form their condensates, lepton number is spontaneously broken and the associated majoron is A_2 [see (6)–(8)]. Note that this massless majoron is phenomenologically acceptable since it will escape detection (i.e., will decouple from the model) due to its extremely suppressed couplings to the matter and gauge fields [13].

As opposed to our light seesaw model, in the Calmet model the neutrinos will acquire only Dirac masses through interactions of the superheavy $SU(2)$ doublet Φ_2 with the neutrinos, i.e., $\mathcal{L}_{m_\nu} = Y_{\rm D}\ell_{\rm L}\Phi_2\nu_{\rm R}$. In this model the "double seesaw" mechanism required for generating the sub-eV neutrino masses is operational already in the scalar sector. That is, when Φ_2 forms its "double seesawed" superlight condensate $\langle\Phi_2\rangle \sim (2\lambda_1)^{-1/2} \times \Lambda_{\rm I}^4/\Lambda_{\rm U}^3$ [see (10)] the neutrino acquires a Dirac mass of that order:

$$m_\nu^{\rm D} = Y_{\rm D}\langle\Phi_2\rangle \sim \frac{Y_{\rm D}}{\sqrt{2\lambda_1}}\frac{\Lambda_{\rm I}^4}{\Lambda_{\rm U}^3} \sim \frac{Y_{\rm D}}{\sqrt{2\lambda_1}}\frac{\Lambda_{\rm EW}^2}{\Lambda_{\rm U}}. \qquad (14)$$

In particular, if $Y_{\rm D} \sim \lambda_1 \sim \mathcal{O}(1)$, then $m_\nu^{\rm D} \sim \mathcal{O}(10^{-3})\,{\rm eV}$ for $\Lambda_{\rm I} \sim \mathcal{O}(10^9)\,{\rm GeV}$ and $\Lambda_{\rm U} \sim \mathcal{O}(10^{16})\,{\rm GeV}$ [which also guarantees that $\Lambda_{\rm EW} \sim \Lambda_{\rm I}^2/\Lambda_{\rm U} \sim \mathcal{O}(100)\,{\rm GeV}$]. However, if $\Lambda_{\rm U} \sim \mathcal{O}(10^{19})\,{\rm GeV}$ [which requires $\Lambda_{\rm I} \sim \mathcal{O}(10^{10.5})\,{\rm GeV}$ in order to reproduce the EW scale] it is difficult to see in the Calmet model how $\mathcal{O}(10^{-3})\,{\rm eV}$ neutrino masses can be generated, unless we assign a rather un-naturally large value to $Y_{\rm D}$, i.e., $Y_{\rm D} \sim \mathcal{O}(10^3)$.

Note also that, in the Calmet model, a Dirac type neutrino mass of the order of $\Lambda_{\rm EW}$ would be generated if the light Higgs doublet Φ_1 is also coupled to the right-handed neutrinos via $Y_{\rm D}\ell_{\rm L}\Phi_1\nu_{\rm R}$. Therefore, in order to protect the sub-eV neutrino mass scale one has to assume that such $\ell_{\rm L}\Phi_1\nu_{\rm R}$ interactions are absent.

4 Seesaw Higgs mechanism from tiny extra dimensions

The idea that the enormous hierarchy between the two seemingly disparate EW and Planck scales may result from the existence of compact extra spatial dimensions (CED) [5], has gained intense interest in the past years, due to its novel approach. In particular, according to the viewpoint taken in [5], the EW scale is the only fundamental scale in nature and the effective large four-dimensional Planck scale (i.e., the weakness of gravity) is a result of the large size of the CED (the bulk), through which gravity propagates:

$$R \sim \frac{1}{M_\star}\left(\frac{M_{\rm Pl}}{M_\star}\right)^{2/n}, \qquad (15)$$

where R is the typical radius of the CED, n is the number of CED, $M_{\rm Pl} \sim 10^{19}\,{\rm GeV}$ is the reduced four-dimensional Planck mass and $M_\star$ is the fundamental $(4+n)$ Planck scale. Putting $M_\star = \Lambda_{\rm EW}$ in (15) implies that R is in the sub-mm range.

In contrast to [5], let us suppose that the $(4+n)$ fundamental Planck scale is close to the GUT scale, i.e., $M_\star \sim 10^{16}\,{\rm GeV} >> \Lambda_{\rm EW}$ (note that this is similar to the view taken in the "warp" extra-dimension scenario [6]). In this case, the typical size of the bulk CED where gravity propagates is extremely small $R(M_\star \sim 10^{16}\,{\rm GeV}) = 10^{\frac{4}{n}-17}\,{\rm fm}$, and so, present and future searches for deviations from Newtonian gravity are clearly hopeless.

Although such a scenario with "tiny" extra dimensions (TED) may seem phenomenology unattractive, it turns out that in the context of our light seesaw model, the seesaw induced EW scale is naturally obtained if indeed the CED are tiny and the fundamental scale is $\Lambda_{\rm U} = M_\star \sim 10^{16}\,{\rm GeV}$. Thus, in what follows we will consider a specific mechanism, based on models with TED, that can generate the desired intermediate scale $\Lambda_{\rm I} \sim 10^9\,{\rm GeV}$ on our brane, which then triggers the seesaw Higgs mechanism (when the fundamental scale is taken to be the GUT scale).

Let us suppose that the CED are populated with multiple 3-branes. In this case, it was shown in [14] that the violation of flavor symmetries on these distant branes can be carried out to our brane by "messenger" scalar fields that can propagate freely in the bulk between the branes. In particular, after propagating through the bulk's CED (from the distant brane to our world), the profile of these messenger fields at all points on our wall (i.e., on the interference between the bulk and our brane) "shines" the flavor violation which appears as a boundary condition on our 3-brane. This mechanism may explain e.g., the smallness of some Yukawa couplings and the sub-eV mass scale of the neutrinos [14–16]. In our case, this "shining" mechanism is utilized to generate the light seesaw Higgs model presented in Sect. 2.1. Somewhat similar to [15], we will assume that a messenger singlet scalar field "shines" only the scalar sector in our brane.

Let η be a singlet scalar field which can propagate in the CED and let $\eta_{\mathcal{P}'}$ be another singlet localized to a distant 3-brane ($\mathcal{P}'$-brane), which is situated at $y^i = y_0^i$ in the CED (y^i, $i = 1$–n, is the coordinates of the CED such that our brane is assumed to be localized at $y^i = 0$). In particular, we assume that $|y_0| = R$ (R is the CED radius) which is the farthest point in the CED. Suppose now that some "scalar-flavor" symmetry G_S is initially conserved everywhere and that both η and $\eta_{\mathcal{P}'}$ are charged under G_S, with charges $S_\eta = -S_{\eta_{\mathcal{P}'}} = 1$. In particular, these two singlets interact on the $\mathcal{P}'$-brane via

$$S_{\mathcal{P}'} = \int_{\mathcal{P}'} \mathrm{d}^4 x' M_\star^2 \eta_{\mathcal{P}'}(x') \eta(x', y^i = y_0^i), \tag{16}$$

where x' is the coordinate in the $\mathcal{P}'$-brane (recall that $M_\star$ is the fundamental Planck scale).[3]

Then, when $\eta_{\mathcal{P}'}$ acquires a non-zero VEV in the $\mathcal{P}'$-brane, the "scalar-flavor" S number is spontaneously violated and $\langle \eta_{\mathcal{P}'} \rangle$ will act as a point source, shining the η field. The S number violation will then be communicated to our world through the shined value of $\langle \eta \rangle$ on our wall [14]:

$$\langle \eta(x, y^i = 0) \rangle = \langle \eta_{\mathcal{P}'}(y^i = y_0^i) \rangle \times \Delta_n(R = |y_0|), \tag{17}$$

where x^μ ($\mu = 0$–3) are the usual non-compact dimensions of our brane and Δ_n is the Yukawa potential in the n transverse dimensions [14]. For example, for $n > 2$, $m_\eta R << 1$ and $\langle \eta_{\mathcal{P}'} \rangle \sim M_\star$, one obtains the following profile of shining on our brane [14–16]:

$$\langle \eta \rangle \sim \frac{\Gamma\left(\frac{n-2}{2}\right)}{4\pi^{\frac{n}{2}}} \frac{M_\star}{(M_\star R)^{n-2}}, \tag{18}$$

which now appears as a boundary condition for our four-dimensional scalar potential.

Consider now the light seesaw scalar potential in (4) and assume that the dimension-two operator $\varphi^\dagger \chi$ carries an S number -2, i.e., $S_{\varphi^\dagger \chi} = -2$. If G_S is initially conserved everywhere, i.e., also in the bulk and on our brane, then η interacts on our brane via the following S number conserving term:

$$S_{us} = \int_{us} \mathrm{d}^4 x \; \eta(x, y^i = 0) \eta(x, y^i = 0) \varphi^\dagger(x) \chi(x) + \mathrm{h.c.} \tag{19}$$

Thus, after propagating the distance between the $\mathcal{P}'$-brane to our brane, the shined value of η, $\langle \eta \rangle \equiv \langle \eta(x, y^i = 0) \rangle$, will generate the term $\Lambda_\mathrm{I}^2 \mathrm{Re}(\varphi^\dagger \chi)$ in (4), if $\langle \eta \rangle = \Lambda_\mathrm{I}$. Using (18) with $M_\star \sim 10^{16}$ GeV and with $M_\star R \sim (M_\mathrm{Pl}/M_\star)^{2/n}$ from (15), the desired (i.e., in order to get the seesaw induced EW scale) intermediate scale, $\langle \eta \rangle = \Lambda_\mathrm{I} \sim 10^9$ GeV, is obtained if there are $n = 7$ tiny extra transverse dimensions of size $R \sim M_\star^{-1} \sim 10^{-16}$ fm. The mass of the messenger field is bounded by $m_\eta << R^{-1}$ and so it can be naturally of the same order of its shined VEV, i.e., $m_\eta \sim \langle \eta \rangle = \Lambda_\mathrm{I} \sim 10^9$ GeV.

[3] We ignore possible self-interaction terms of η in the bulk.

5 Summary

We have proposed a scalar model – the "light seesaw" model – in which the EW scale Λ_EW, is generated through a seesaw Higgs mechanism in the scalar sector. Assuming that the fundamental scale is close to the GUT or Planck scale $\Lambda_\mathrm{U} \sim 10^{16}$–$10^{19}$ GeV, this model is constructed at an intermediate high scale $\Lambda_\mathrm{I} \sim 10^9$–$10^{10.5}$ GeV, at which the SM $SU(2)_L \times U(1)_\mathrm{Y}$ gauge symmetry is spontaneously broken by a SM-like Higgs condensate $\langle \Phi \rangle = \Lambda_\mathrm{I}^2/\Lambda_\mathrm{U} \sim \Lambda_\mathrm{EW}$. The intermediate scale Λ_I is viewed as the scale of breaking of the unification group that underlies the physics at Λ_U.

The model proposed is minimally constructed in the sense that a minimal scalar sector which manifests the seesaw Higgs mechanism was assumed, along with the usual SM gauge and matter fields and with only the addition of right-handed neutrino fields to account for the recently verified non-zero neutrino masses.

We have shown that our light seesaw model has two main achievements.
(1) It naturally explains the huge gap or desert between the fundamental scale Λ_U and the EW scale Λ_EW, by a Λ_U–Λ_I seesaw structure of the scalar potential that sets $\Lambda_\mathrm{EW} \sim \Lambda_\mathrm{I}^2/\Lambda_\mathrm{U}$. The hierarchy problem of the SM is, therefore, alleviated since the EW scale is not fundamental but is rather generated in terms of ultra-high energy phenomena.
(2) It successfully predicts the existence of sub-eV neutrino masses through a "two-step" seesaw mechanism; the first in the scalar sector, $\Lambda_\mathrm{EW} \sim \Lambda_\mathrm{I}^2/\Lambda_\mathrm{U}$, and the second in the neutrino sector, $m_\nu \sim \Lambda_\mathrm{EW}^2/\Lambda_\mathrm{U} \sim \Lambda_\mathrm{I}^4/\Lambda_\mathrm{U}^3$.

Thus, putting $\Lambda_\mathrm{U} \sim 10^{16}$ GeV and $\Lambda_\mathrm{I} \sim 10^9$ GeV or $\Lambda_\mathrm{U} \sim 10^{19}$ GeV and $\Lambda_\mathrm{I} \sim 10^{10.5}$ GeV, our model naturally explains the simultaneous existence of the three very disparate scales we observe in nature: $\Lambda_\mathrm{U} \sim M_\mathrm{GUT} - M_\mathrm{Planck} \sim 10^{16}$–$10^{19}$ GeV, $\Lambda_\mathrm{EW} \sim M_W \sim \mathcal{O}(100)$ GeV and $\Lambda_\nu \sim m_\nu \sim \mathcal{O}(10^{-2}$–$10^{-3})$ eV, at the expense of introducing the intermediate physical scale $\Lambda_\mathrm{I} \sim 10^9$ GeV.

Furthermore, we have shown that the mechanism of a seesaw induced EW scale may naturally emanate from models with *tiny* extra spatial dimensions of size $R \sim M_\star^{-1}$, where $M_\star \sim \Lambda_\mathrm{U} \sim 10^{16}$ GeV is the fundamental multi-dimensional Planck scale. In particular, the existence of numerous *tiny* extra dimensions can generate an intermediate scale of the right size (i.e., $\Lambda_\mathrm{I} \sim 10^9$ GeV), which then triggers the seesaw Higgs mechanism and induces the many orders of magnitudes smaller EW scale. More specifically, we found that our light seesaw scalar potential can be generated when a violation of some flavor symmetry at a distant brane is carried out to our brane by messenger bulk scalar fields (i.e., the "shining" mechanism [14]), if there are seven *tiny* compact extra dimensions of size $R \sim 10^{-16}$ fm.

While our light seesaw model successfully addresses the hierarchy problem of scales as well as naturally explains the observed sub-eV mass scale of the neutrinos, we emphasize that this model requires a UV completion. Although we have not discussed the details of a theory that can underlie such seesaw Higgs models, we have outlined some of its salient features. In particular, the technical aspect of naturalness[4] requires that such seesaw Higgs models be embedded into a more symmetric grand unified theory such that the stabilization of the $\mathcal{O}(\Lambda_{\rm EW})$ mass of the light Higgs boson is ensured by some higher symmetry at the fundamental scale $\Lambda_{\rm U}$. For example, a high-scale supersymmetric framework or scale invariance properties of the GUT or Planck scale physics (as suggested in [4]) may protect the EW Higgs mass scale from fine tuning.

Finally, we note that the experimental signatures (e.g., at the LHC) of the light seesaw model will be similar to those of the SM and will, therefore, stand in contrast to the expectations from e.g., SUSY theories, in which several scalars with masses smaller than $\mathcal{O}$(TeV) should be observed.

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[4] It should be understood, however, that since the EW-scale is not fundamental in the framework of seesaw Higgs models, the naturalness or fine tuning problem has nothing to do with the hierarchy of mass scales, but rather, it is just a technical obstacle that reflects our ignorance regarding the underlying physics at energies above $\Lambda_{\rm I}$.